Analyzing the throughput of wireless unicast networks with output feedback

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Abstract. Wireless communication has improved drastically over the last decade. The wireless revolution, which began in the 1990s, started a paradigm shift from wired to wireless technology. While wired networks will always be needed for their better performance and higher reliability, wireless communication has become more popular due to their portability and ease of use.

Wireless communication can be found at every scale, such as WiFi for short-range communication, all through satellite networks for global systems.

While wireless networks are broadcasting in nature, most of our communication happens in unicast mode, meaning that the message sent by the transmitter is addressed to only one receiver. This significantly decreases the throughput of our network when multiple receivers are connected to the same transmitter.

On the other hand, wireless networks are usually classified as unreliable because of their packet loss. While the original recipient may not correctly receive the message sent by the transmitter, other receivers might. Using these packets, fewer messages might be needed to complete the transfer. For example, let's suppose that in our network, A received B's packet, and B received A's packet. In this case, sending a combination of both packets (such as A+B), both receivers could reconstruct their original package (supposing they receive this combination).

In this paper, I evaluate the advantages of this method in a network containing two receivers by using a Markov chain to simulate the state of the system.

Keywords: wireless unicast network; WiFi; output feedback; network coding; Markov chain

1 Introduction

The first wireless network was developed in 1969 (which came online in 1971) under the name AlohaNET. While broadcasting applications, such as TV and radio stations, have already used wireless transmission due to their ease of use, *unicast* (one-to-one) communication up to this point used wired communications. While the throughput was significantly lower than the wired counterparts (the terminals connected to them were rated at 9600 bps, while Ethernet could reach up to 3 Mbps), this network established a connection between remote islands and helped pave the way for future generations of wireless data communication.

Advancements in MOSFET technology in the 1990s helped the adoption of wireless technologies in consumer applications. While the first commercial wireless network was released in 1986, one of the first major wireless technology was the second generation of cellular networks (abbreviated for *2G*), which provided digitally encrypted mobile phone conversations as well as data services, such as SMS text messages, and later on even rudimentary access to the Internet. Later improvements (3G, 4G, LTE) brought significant improvements in available functions and network throughput.

In this paper, I want to investigate the improvements in network efficiency with the use of broadcasting communication. Even though the packet might not reach the intended receiver (due to environmental noises), other devices in the network could read this message. Using these messages, in certain situations, the sender would only have to resend one packet containing a combination of the lost messages in order for every receiver to reconstruct their message, instead of resending every lost packet. I will examine this strategy in one transmitter, two receiver setup. I will also provide a model to analytically calculate the exact network efficiency of a well-defined strategy instead of using simulations.

The rest of the paper is organized as follows: Section 2 summarizes and presents the most important research areas in the field of wireless unicast networks with output feedback. Section 3 presents the model of the framework. Section 4 shows the solution to calculate the efficiency of a well-defined routing strategy. Section 5 summarizes the results and describes my further research plan.

2 Related work

Network coding was introduced by Ahlswelde et al. in 2000 as a new field in information theory [1]. Contrary to traditional views, Network Coding combines different information flows to improve network throughput, security, and robustness against network losses [2]. While typically used in networks with multiple sinks and sources due to the fact that intermediary nodes can create new coded packets from previously received ones, it can also be used as an erasure code over erasure channels.

In particular, Wang focuses on the network capacity of a 1-to-k broadcast packet erasure channel (PEC) with channel output feedback [3]. The results show significant improvements over traditional packet resending, with increased network throughput and network reliability. He introduces a new class of intersession network coding schemes, termed Packet Evolution. With this scheme, he shows the general region capacity of a 1-to-3 broadcast PEC, the region capacity of two particular cases of 1-to-k broadcast PEC, and the general outer and inner bounds of capacity.

Katti et al. proposed a new architecture for wireless mesh networks called COPE [4]. They increased the network throughput by intelligently mixing packets. After implementing the architecture in a testing environment consisting of 20 nodes spanning two levels of a building, they found that the theoretical gains translated well to real-life applications, improving Internet speeds between 5% and 70% depending on the ratio of download traffic to upload traffic.

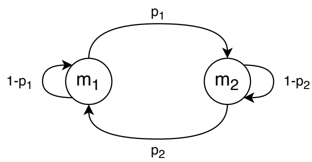

Figure 1: Markov-chain of a simple router

3 System model

The goal of this paper is to provide an analytical solution for calculating the efficiency of a network with a given packet sending strategy. To accomplish this, I first define the model in which I developed the strategy.

The network consists of 1 sender node (called *router* due to the similarities in a WiFi setup) and k receiver node. In this network, the router sends a packet and receives feedback about which nodes failed to read the message. Suppose that the rate with which the router can send a packet to every receiver is constant. When a packet is sent, the probability that node i receives the packet is p_i, which does not change during different packet transfers.

Every receiver wants a unique flow of messages, meaning that one message is only addressed to one receiver. The router broadcasts packets, which may be one message (hoping that the intended receiver will successfully receive this message), or it may be the combination of several different messages. In this paper, network coding over $GF(2)$ is used to combine different messages, but any other erasure code may be used. In this case, the combination of different messages is the XOR combination of them, denoted by $\oplus$.

Unless otherwise stated, m_i denotes a message addressed to node i. Furthermore, for ease of use, $m_i^\star$ denotes the last m_i which was not received by node i.

The router decides which messages to combine based on which previous packets each receiver has (for example, if receiver *1* has $m_2^\star$, and receiver *2* has $m_1^\star$, it might send $m_1^\star \oplus m_2^\star$). To model this, I decided to limit the number of feedback a router may store to a pre-defined constant C. Due to this, we can define a finite number of states the router may be depending on the feedbacks stored.

A well-defined routing strategy contains the definition of every state, and for every state, the next packet to be sent. This can be modeled with a Markov-chain, where the transitions are based on which receiver receives the sent packet. For example, figure 1 shows the Markov-chain of a simple router which alternates between two receivers when the intended receiver receives the sent packet. The messages in the states show what the next packet should contain.

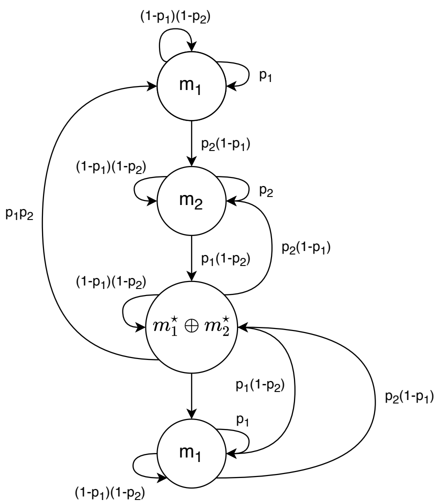

Figure 2: Routing strategy for two receivers

4 Calculating the goodput of a network

After defining the system model, I developed a solution to calculate the efficiency of a network with a well-defined routing strategy. In a simulated environment, this is usually calculated by running the simulation for a longer time and dividing the number of received packets with the number of sent packets. To explain the algorithm, I present the strategy in a 1-to-2 network shown in figure 2.

In this strategy, we first start by sending messages to receiver 1. When receiver 1 does not receive the packet, but receiver 2 does, we switch to sending messages to receiver 2. When receiver 2 does not receive a packet, but receiver 1 does, we step to the next state. In this situation, both receivers are in possession of a message intended for the other receiver. If we send the combination of these messages, no matter which receiver receives the packet, they will be able to calculate their own message. Based on this observation, if only one receiver receives this combination, we start sending to it again until there is a packet which only the other receiver receives, after which we send the combination of the last two lost message again; while if both of them receive the packet, we jump back to the starting state. If, in any state, none of the receivers receive the next packet, we stay in the same state and resend the same message.

To calculate the number of messages sent and received by each node, we have to convert the Markov-chain to a form where these properties are attached to the states. For this, I add a state to every edge containing the number of decoded (useful) messages and the total amount of sent messages as can be seen on figure 3. The conversion follows these rules:

- The states in the original chain have 0 received and 0 sent message.
- For every edge, the state belonging to the edge has 1 sent message.

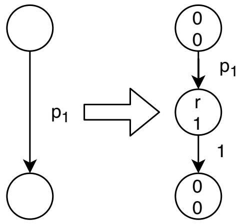

Figure 3: Converting edges from the Markov-chain

- If the sent packet is not a combination, the received message count is 1 if the intended receiver received the message, 0 otherwise.
- If the sent packet is a combination of messages, the received message count is the number of receivers capable of decoding their message with this combination.

If neither p_1 or p_2 is 0 or 1, the resulting Markov-chain will be aperiodic and positive recurrent, making it ergodic. Because of this, this Markov-chain will have a unique stationary distribution. Knowing this distribution, we can calculate the efficiency of the network as the ratio of the received and sent messages based on the states weighted by the stationary distribution.

After inputting the converted Markov-chain, the following equations need to be solved, where M is the matrix form of the chain:

$$xM = x \tag{1}$$

$$sum(x) = 1 \tag{2}$$

Having solved for x, we can calculate the efficiency with the following equation:

$$efficiency = \frac{w_r x}{w_t x} \tag{3}$$

, where w_r and w_t are the received and total message count vectors obtained from the conversion.

For the special case when $p_1 = p_2 \equiv p$, we get $2p\frac{p-2}{p-3}$ for the efficiency ratio, which is equivalent to the results found in other literature [3]. Figure 4 shows the efficiency ratio compared to the simple routing under different p values.

5 Conclusion

In this paper, I have shown a novel approach to calculating the efficiency of a given well-defined routing strategy based on the stationary distribution of a Markov-chain. With the help of a routing strategy, I have shown that this solution matches the results of other researches.

With the current algorithm, I have manually converted the routing strategy to the converted Markov-chain due to it's small size. While not trivial, it is

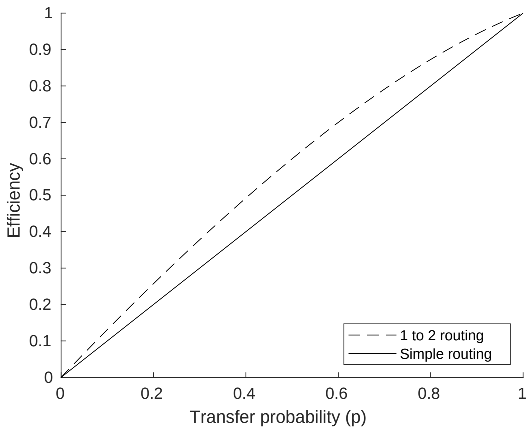

Figure 4: Efficiency of the routing strategies

not impossible to generate the converted chain from the initial strategy, which I would like to develop in the future. Using this analytical solution, there is no need to run any kind of simulation, making it efficient for testing other efficient routing strategies.

Acknowledgments

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